

## Fixed Point Iterations

Banach-Picard  $x^{k+1} = T(x^k)$

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Krasnosel'skiĭ-Mann  $x^{k+1} = x^k + \alpha_k [T(x^k) - x^k]$

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Fast Krasnosel'skiĭ-Mann 
$$\begin{aligned} x^{k+1} &= x^k + \frac{\alpha}{2(k+\alpha)} [T(x^k) - x^k] \\ &\quad + \frac{k}{k+\alpha} [T(x^k) - T(x^{k-1})] \end{aligned}$$

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Heavy-ball  $x^{k+1} = T(x^k + \alpha_k [x^k - x^{k-1}])$

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Halpern  $x^{k+1} = \alpha_k u + (1 - \alpha_k) T(x^k)$

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Viscosity Approximation  $x^{k+1} = \alpha_k f(x^k) + (1 - \alpha_k) T(x^k)$

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Ishikawa 
$$\begin{cases} y^k = x^k + \beta_k [T(x^k) - x^k] \\ x^{k+1} = (1 - \alpha_k)x^k + \alpha_k T(y^k) \end{cases}$$



## References

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